

BRANS-DICKE WORMHOLE AND SPACE-TIME FOAM

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We introduce, by means of the Brans-Dicke scalar field, space-time fluctuations at scale comparable to Planck length near the event horizon of a black hole and examine their dramatic effects.

It is generally believed that the space-time continuum will be replaced by a more fundamental level of description at length scales smaller than the Planck length l_P , where the classical description breaks down to a “quantum foam” [1] and the space-time topology fluctuates over a spatial distance comparable with l_P .

Moreover in recent papers Amelino-Camelia [2] and Ng and van Dam [3], basing on a gedanken timing experiment originally devised by Salecker and Wigner [4], argue that such a distance could be much greater than Planck length. As a consequence quantum gravity effects could be probed with current or future interferometers designed for the gravitational-wave detectors.

In this communication, starting from a different point of view, we give some arguments which can lead, already when distances vary at the Planck scale, to dramatic effects near the event horizon of the Schwarzschild vacuum solution now perturbed by the corresponding fluctuations of space-time. We will show that the metric breaks down, even for large masses, in correspondence of the surface which should have been the event horizon, so it becomes impossible not only to define an event horizon but even to foresee which singularity has replaced it. These facts signal the need of a fully quantum description of gravity in the presence of distances suitable for the Planck regime even if surfaces are much greater than l_P^2 and the gravitational field in their neighbourhood is weak.

To achieve this goal we start by considering the static spherically symmetric vacuum solution of the Brans-Dicke theory of gravitation [5].

The related calculations were performed by us in Ref. [6], working in the Jordan frame, where the action is given (in units $G_0 = c = 1$) by

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[\Phi R - \frac{\omega}{\Phi} \nabla^\alpha \Phi \nabla_\alpha \Phi \right] \quad (1)$$

and with a suitable choice of gauge. Here we quote only the results relevant for the following.

The line element can be written as

$$ds^2 = e^{\mu(r)} dr^2 + R^2(r) d\Omega^2 - e^{\nu(r)} dt^2 \quad (2)$$

where $d\Omega^2 = d\vartheta^2 + \sin^2 \vartheta d\varphi^2$ and, in the selected gauge:

$$R^2(r) = r^2 \left[1 - \frac{2\eta}{r} \right]^{1-\gamma\sqrt{2/(1+\gamma)}} \quad (3a)$$

$$e^{\mu(r)} = \left[1 - \frac{2\eta}{r} \right]^{-\gamma\sqrt{2/(1+\gamma)}} \quad (3b)$$

$$e^{\nu(r)} = \left[1 - \frac{2\eta}{r} \right]^{\sqrt{2/(1+\gamma)}} \quad (3c)$$

Here γ is the post-Newtonian parameter

$$\gamma = \frac{1 + \omega}{2 + \omega} \quad (4)$$

and

$$\eta = M \sqrt{\frac{1 + \gamma}{2}} \quad (5)$$

Finally the scalar field is given by

$$\Phi(r) = \Phi_0 \left[1 - \frac{2\eta}{r} \right]^{(\gamma-1)/\sqrt{2(1+\gamma)}} \quad (6)$$

while the effective gravitational coupling $G(r)$ equals

$$G(r) = \frac{1}{\Phi(r)} \frac{2}{(1 + \gamma)} \quad (7)$$

the factor $2/(1 + \gamma)$ being absorbed, as in Ref. [5], in the definition of G .

Departures from Einstein's theory of General Relativity appear only if $\gamma \neq 1$, a possibility consistent with experimental observations which estimate it in the range $1 - 0.0003 < \gamma < 1 + 0.0003$ corresponding to the dimensionless Dicke coupling constant $|\omega| > 3000$.

When $\gamma < 1$ and $r \rightarrow 2\eta$, then $R(r)$, $e^{\nu(r)}$ and $G(r)$ go all to zero. Therefore we have a singularity with infinite red-shift and gravitational interaction decreasing while approaching the singularity.

When $\gamma > 1$, the null energy condition (NEC) is violated [7] and a wormhole solution is obtained [6] with throat at

$$r_0 = \eta \left[1 + \gamma \sqrt{\frac{2}{1 + \gamma}} \right] = M \left[\gamma + \sqrt{\frac{1 + \gamma}{2}} \right] \quad (8)$$

to which corresponds the value R_0 given by equation (3a). It is easy to verify that $r_0 > 2\eta$ and $R_0 > 0$. Now the singularity is beyond the throat and is smeared on a spherical surface, asymptotically large but not asymptotically

flat, of radius $R(r) \rightarrow \infty$ as $r \rightarrow 2\eta$ and where also the red-shift and $G(r)$ become infinitely large [8,9].

When $\gamma = 1$ exactly, one has Schwarzschild solution of General Relativity

$$ds^2 = \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2 - (1 - \frac{2M}{r}) dt^2 \quad (9)$$

where it appears a black-hole with event horizon at $R(r) = r = 2M$.

If future measurements will establish that $\gamma \neq 1$ and if the Jordan frame is the physical frame, then the strong equivalence principle is violated (due to exotic matter in the case $\gamma > 1$) and it will be possible to decide on the type of singularity occurring. Moreover gravitation shall be better described by a suitable generalization of Einstein's theory.

Here we first assume $\gamma = 1$ exactly but then we introduce a fluctuation in the Schwarzschild metric by imposing a violation of the strong equivalence principle, of the order of the Planck length l_P , which comes from the following variation of the gravitational radius :

$$2\eta = 2M \pm \frac{l_P}{2} \quad (10)$$

Making use of Equation (5) one obtains for γ , at first order in l_P/M

$$\gamma = 1 \pm \frac{l_P}{M} \quad (11)$$

so from Equation (4) the dimensionless coupling constant ω turns out to be $|\omega| \approx M/l_P$. To make an example, in the case of a solar mass M_\odot this would amount to a fluctuation of $\Delta M_\odot/M_\odot \approx 10^{-38}$.

It may be useful to rewrite, in this approximation, the metric coefficients of the Brans-Dicke line element:

$$R^2(r) = r^2 \left[1 - \frac{2M \pm \frac{l_P}{2}}{r} \right]^{\mp \frac{3l_P}{4M}} \quad (12a)$$

$$e^{\mu(r)} = \left[1 - \frac{2M \pm \frac{l_P}{2}}{r} \right]^{-1 \mp \frac{3l_P}{4M}} \quad (12b)$$

$$e^{\nu(r)} = \left[1 - \frac{2M \pm \frac{l_P}{2}}{r} \right]^{1 \mp \frac{l_P}{4M}} \quad (12c)$$

Here the upper sign refers to the case $\gamma > 1$ and the lower one to the case $\gamma < 1$. It is apparent that, while in the case $\gamma = 1$ the event horizon is fixed at

$r_S = 2M$, in the presence of space-time fluctuations one may have, taking the extreme values of γ given by Equation (11), either a naked singularity at $r_< = 2M - l_P/2$ if $\gamma = 1 - l_P/M$ or a wormhole with a throat at $r_0 = 2M + (5/4)l_P$ if $\gamma = 1 + l_P/M$; in this latter case we have beyond the throat at $r_> = 2M + l_P/2$ another singularity which is asymptotically large but not asymptotically flat. We recall that $r_<$ and $r_>$ are the limits between which we required to fluctuate the metric. When r exceeds $2M$ by several units of l_P the solution of the field equations behaves as in the Schwarzschild case, but when r varies around $2M$ in an interval containing all the possible singularities one is faced with different interchanging behaviours which are unpredictable on classical grounds. In this latter case the proper distance from the singularity of a point near to $2M$ is proportional to $\sqrt{Ml_P}$. To make a numerical example in the case $\gamma > 1$, the proper radial distance of $r_>$ (to which corresponds an infinite value of the standard radial coordinate R) from r_0 (to which corresponds the location R_0 of the throat) is

$$L = \int_{r_>}^{r_0} e^{\mu(r)/2} dr \approx 2.51 \sqrt{Ml_P} \quad (13)$$

and the proper volume comprised between the radii is

$$V = 4\pi \int_{r_>}^{r_0} R^2(r) e^{\mu(r)/2} dr \approx 36\pi \sqrt{M^5 l_P} \quad (14)$$

Numerical values for a solar mass are $L = 3.9 \cdot 10^{-16} m$ and $V = 3.4 \cdot 10^{-10} m^3$. So, if it is correct to introduce space-time fluctuations by means of the Brans-Dicke scalar field, then it would be needed a full quantum theory of gravitation to describe not only the kind and the evolution of the singularities but also the space-time that surrounds them at distances much greater than the Planck length. If the scenario we have proposed here is realistic, it should turn out to be a good candidate both as a source of strong gravity waves and as a central engine required for the production of gamma ray bursts.

References

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